Exercise 34

(a) If $f(x) = x + 1/x$, find $f'(x)$.

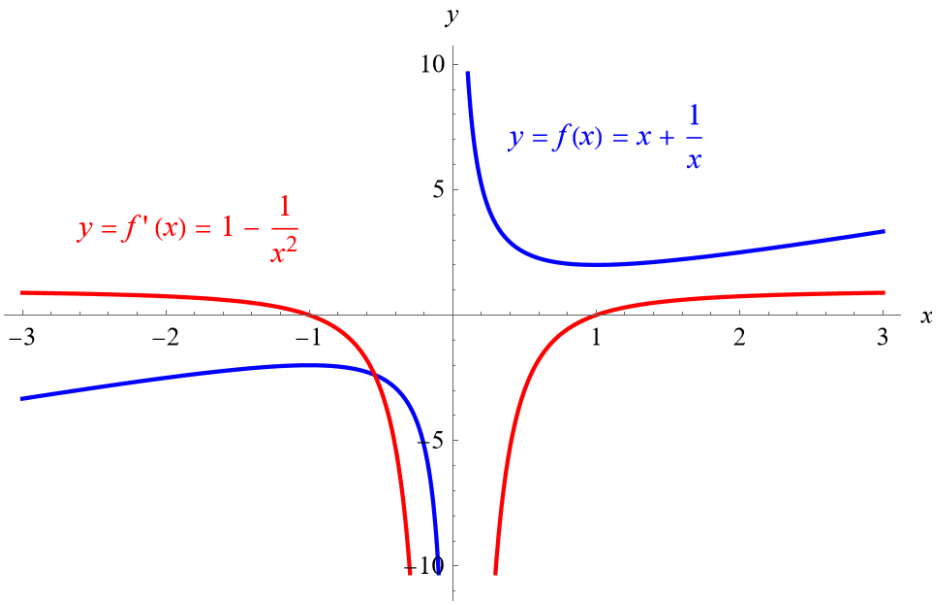
(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

Solution

Calculate the derivative of $f(x)$ using the definition.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\left[(x+h) + \frac{1}{x+h}\right] - \left(x + \frac{1}{x}\right)}{h} \\
&= \lim_{h\to 0} \frac{h + \frac{1}{x+h} - \frac{1}{x}}{h} \\
&= \lim_{h\to 0} \frac{h + \frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{h + \frac{x-(x+h)}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \frac{h + \frac{-h}{x(x+h)}}{h} \\
&= \lim_{h\to 0} \left[\frac{h}{h} + \frac{-h}{hx(x+h)}\right] \\
&= \lim_{h\to 0} \left[1 - \frac{1}{x(x+h)}\right] \\
&= 1 - \frac{1}{x(x)} \\
&= 1 - \frac{1}{x^2}
\end{aligned}$$

Below is a graph of $f(x)$ and $f'(x)$ versus x.


y
10
5
−5
−10
x
−3
−2
−1
1
2
3
$y = f(x) = x + \frac{1}{x}$
$y = f'(x) = 1 - \frac{1}{x^2}$